Homework for the course of *Integer Linear Programming for Combinatorial Optimization Problems*

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Homework has to be returned by email by the end of August. Please use [ECI2012 - Homework] in the subject. Solutions written by hand and scanned are ok!

1. Propose a greedy algorithm for computing a maximal clique in a given graph.

2. Consider a 2D-Vector Packing Problem, that is, a KP01 problem with two independent capacity constraints. The problem can be interpreted as follows: each item i has a profit p_i, a weight w_i and a volume v_i, $i = 1, \dots, n$; and one has to select a subset of items of maximum profit without exceeding a weight capacity W and a volume capacity V.

   Give an integer linear model for the problem, and then relax the second capacity constraint in a Lagrangian fashion. How would you apply an algorithm for the KP01 problem to the resulting problem?

3. Given an undirected graph $G = (V, E)$ consider a weighted Vertex Coloring Problem, in which a positive weight w_i is associated to each vertex $i \in V$. The problem is to assign a color to each vertex in such a way that adjacent vertices receive different colors, and the objective is to minimize the sum of the costs of the colors used, where the cost of each color is given by the maximum weight of the vertices assigned to that color (whereas in the traditional Vertex Coloring Problem the aim is to minimize the number of colors used). For example, if $w_1 = 3$, $w_2 = 4$, $w_3 = 1$, a solution coloring 1 and 3 with a first color and 2 with a second color would cost $max\{3, 1\} + 4 = 7$, while a solution coloring 1 and 2 with a first color and 3 with a second color would cost $max\{3, 4\} + 1 = 5$.

   i) Propose a "descriptive" model for the problem (similar to model (1) for the vertex coloring);

   ii) Propose a Set-Covering formulation of the problem, with a suited definition of the variables and their cost.

   iii) Discuss how to tackle the linear relaxation of this formulation via column generation.

4. Consider the "descriptive" model for the Cutting Stock Problem defined at page 10 of the lecture on cutting. Show how to apply the Dantzig-Wolfe reformulation so as to obtain the corresponding Set-Covering formulation of page 14.

5. Model M2 by Lodi and Monaci (page 33 of the lecture on cutting) is derived from model M1 by distinguishing items with respect to shelves initialization. Exploit the same ideas to derive a model for the Cutting Stock Problem, by improving model for the 2DBPP by Lodi, Martello and Vigo (page 37).